Turbulent equipartition theory of toroidal momentum pinch[a)]

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The mode-independent part of the magnetic curvature driven turbulent convective (TurCo) pinch of the angular momentum density [Hahm *et al.*, Phys. Plasmas **14**, 072302 (2007)], which was originally derived from the gyrokinetic equation, can be interpreted in terms of the turbulent equipartition (TEP) theory. It is shown that the previous results can be obtained from the local conservation of "magnetically weighted angular momentum density," $nm_iU_\parallel R/B^2$, and its homogenization due to turbulent flows. It is also demonstrated that the magnetic curvature modification of the parallel acceleration in the nonlinear gyrokinetic equation in the laboratory frame, which was shown to be responsible for the TEP part of the TurCo pinch of angular momentum density in the previous work, is closely related to the Coriolis drift coupling to the perturbed electric field. In addition, the origin of the diffusive flux in the rotating frame is highlighted. Finally, it is illustrated that there should be a difference in scalings between the momentum pinch originated from inherently toroidal effects and that coming from other mechanisms that exist in a simpler geometry. © *2008 American Institute of Physics*.
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I. INTRODUCTION

Understanding momentum transport, which influences plasma rotation, is very important since it can play a crucial role in reducing turbulence and transport as well as in stabilizing magnetohydrodynamic (MHD) instabilities including the resistive wall mode. In most cases, the toroidal momentum transport from experiments is observed to be anomalous, i.e., higher than neoclassical theory predictions, and is therefore believed to be caused by low frequency, ion gyroradius scale, electrostatic drift wave turbulence, including ion temperature gradient (ITG) mode turbulence and trapped electron mode (TEM) turbulence. For instance, the toroidal momentum diffusivity χ_ϕ was comparable to the ion thermal diffusivity χ_i (see Ref. 1) in Tokamak Fusion Test Reactor (TFTR) experiments, in rough agreement with theoretical predictions based on ITG turbulence.[2]

While the toroidal momentum transport is often described by a diffusion coefficient $\chi_{\phi,\text{eff}}$ alone, there is accumulating evidence that a variety of rotation phenomena of great potential importance cannot be properly characterized by the diffusion coefficient only. This includes the observation of spontaneous toroidal rotation of plasmas in the absence of apparent external torque input.[3–11] Some prefer to call it an "intrinsic rotation."[8] In many cases, rotation profiles are peaked near the axis, even for off-axis deposition, zero torque, or no neutral beam injection (NBI), suggesting the existence of a nondiffusive inward flux of toroidal angular momentum.[12] In addition, recent perturbation experiments on JT60-U[13,14] and NSTX[15] neutral beam heated plasmas showed the need for an "inward pinch term" of angular momentum to match the measured centrally peaked rotation profiles.

Theoretically, one can write an expression for the radial flux of the toroidal momentum as

$$\Pi_\phi = -\chi_\phi \frac{d}{dr}U_\phi + V_{\text{pinch}}U_\phi + S.$$

Here, the nondiffusive component of the turbulence driven radial transport of toroidal momentum[16] includes not only the turbulent convective (TurCo) pinch (V_{pinch}), but also the residual stress (S), which does not depend on the flow explicitly. It should be emphasized that a nondiffusive flux of momentum can be obtained from various physics mechanisms.[17–25] Depending on plasma parameters and configurations, a specific mechanism can be more relevant than others, and sometimes a combination of two or more mechanisms is necessary to reproduce basic features of experiments. For instance, a commonality of spontaneous rotation of plasmas[26] in NBI-free H-mode plasmas is the empirical "Rice" scaling,[5] which states that the rotation at the axis is in the co-current direction, and proportional to the incremental stored energy divided by the plasma current. This scaling is suggestive of a mechanism associated with the ∇P_i-driven $\mathbf{E}\times\mathbf{B}$ shear. It is also of interest to study physics mechanisms for an inward pinch of toroidal angular momentum in the absence of $\mathbf{E}\times\mathbf{B}$ shear, since spontaneous rotation has also been observed in L-mode[4,27] and Ohmically heated (OH) plasmas[6,8] in which the $\mathbf{E}\times\mathbf{B}$ shear effect is expected to be weak.

After the details of a quasilinear derivation from the gyrokinetic equation[28] and turbulent equipartition (TEP) interpretation of the mode-independent part of the turbulent con-

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vective (TurCo) pinch of angular momentum density were published in Ref. 24, we presented[29] a simpler and more intuitive derivation of the TEP pinch based on an *ansatz* of local angular momentum conservation and homogenization. In this paper, we recapitulate the essence of these two different approaches and clarify their relation. This puts the TEP interpretation of the mode-independent part of the TurCo pinch, which was originally derived from the gyrokinetic equation, on a firmer and more transparent theoretical ground.

Our quantitative predictions can be summarized as follows. To the lowest order in r/R_0, with R_0 the major radius at the magnetic axis, the TEP pinch velocity is driven by the magnetic field curvature (or equivalently, ∇B in low-β plasmas), rather than ion thermal effects, and given by

$$V_{\mathrm{Ang}}^{\mathrm{TEP}} \simeq -\frac{2F_{\mathrm{balloon}}}{R_0}\chi_{\mathrm{Ang}} \tag{1}$$

for the angular momentum density $nU_\parallel R$, and

$$V_{\mathrm{Mom}}^{\mathrm{TEP}} \simeq -\frac{3F_{\mathrm{balloon}}}{R_0}\chi_{\mathrm{Mom}} \tag{2}$$

for the parallel momentum density $nU_\parallel$. Here, χ_{Ang} and χ_{Mom} are diffusivities for angular momentum and parallel momentum, respectively. A dimensionless coefficient on the order of unity, F_{balloon} characterizes the "ballooning structure" of the turbulence. For poloidally symmetric, flutelike turbulence intensity, $F_{\mathrm{balloon}}\to 0$. For strongly outward ballooning fluctuations (peaked at the low-B side), as are often found from comprehensive linear kinetic calculations[30] based on profiles from experiments, $F_{\mathrm{balloon}}\simeq 1$,[24] and the pinch is inward in radius. The TEP pinch originates from the fact that magnetic curvature can modify the acceleration of ions along the magnetic field, as can be appreciated from the gyrokinetic equations.[28] When the magnetic curvature $[\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}]$ changes its sign along the B field as one moves from the low-B field (bad curvature) side to the high-B field (good curvature) side, the variation of fluctuation amplitude along the magnetic field (a property of ballooning fluctuations in toroidal geometry) can yield a net acceleration. This symmetry breaking mechanism due to magnetic curvature, alongside the $k_\parallel$-symmetry breaking due to the $\mathbf{E}\times\mathbf{B}$ shear,[22] constitutes the unified "$\mathbf{B}^*$-symmetry breaking" as discussed in Ref. 24.

In this paper, we also discuss theoretical issues that arise when one calculates the turbulence driven radial flux of parallel flow in the rotating frame. In particular, we identify terms in the gyrokinetic equation that lead to the diffusive flux and the momentum pinch, respectively. We demonstrate that the magnetic curvature modification of the parallel acceleration in the nonlinear gyrokinetic equation in the laboratory frame,[28] which was shown to be responsible for the TEP part of the TurCo pinch of angular momentum density in our previous work,[24] is closely related to the Coriolis drift coupling to the perturbed electric field.[23,31,32] We also highlight the origin of the diffusive flux in the rotating frame.

The remainder of this paper is organized as follows. In Sec. II, the standard quasilinear derivation of the TEP part of the TurCo pinch is briefly reviewed with a focus on its insensitivity to the plasma model and key physics assumptions. A simple TEP theory interpretation based on the local magnetically weighted angular momentum conservation is given in Sec. III. In Sec. IV, we present theoretical issues which arise when one formulates the turbulence driven radial flux of parallel flow in the rotating frame. Theoretical issues related to scalings of the momentum pinch are discussed in Sec. V. Conclusions are drawn in Sec. VI.

II. QUASILINEAR DERIVATION OF TEP MOMENTUM PINCH IN TOROIDAL GEOMETRY

In this section, we briefly review the standard quasilinear derivation of the TEP part of the TurCo pinch[24] with a focus on mode-independent key physics. We show that a careful treatment of geometric effects due to nonuniform $\mathbf{B}$ (with nonvanishing curvature $\nabla\times\mathbf{b}$), yields a novel pinch mechanism for parallel momentum and angular momentum densities. We cast the expressions in a form where not only the new momentum pinch terms are clearly identified, but also the underlying approximate conservation laws responsible for the TEP pinch are transparent. A more detailed derivation can be found in Ref. 24. Here, we discuss the essential physics ingredients in a simpler manner.

The following nonlinear electrostatic gyrokinetic equation with proper conservation laws in general geometry is given by Eqs. (19), (21), and (22) of Ref. 28:

$$\frac{\partial F}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\nabla F + \frac{dv_\parallel}{dt}\frac{\partial F}{\partial v_\parallel} = 0, \tag{3}$$

with

$$\frac{d\mathbf{R}}{dt} = v_\parallel\frac{\mathbf{B}^*}{B^*} + \frac{c\mathbf{b}}{e_iB^*}\times[e_i\nabla\langle\langle\delta\phi\rangle\rangle + m_i\mu\nabla B] \tag{4}$$

and

$$\frac{dv_\parallel}{dt} = -\frac{\mathbf{B}^*}{m_iB^*}\cdot[e_i\nabla\langle\langle\delta\phi\rangle\rangle + m_i\mu\nabla B]. \tag{5}$$

Here, the gyrokinetic Vlasov equation (3) is written in terms of the gyrocenter distribution function $F(\mathbf{R},\mu,v_\parallel,t)$, with $\mu \equiv v_\perp^2/2B$, and $\langle\langle\ldots\rangle\rangle$ denotes an average over the gyrophase. $\mathbf{B}^*$ is defined by

$$\mathbf{B}^* \equiv \mathbf{B} + \frac{m_ic}{e_i}v_\parallel\nabla\times\mathbf{b}.$$

We can derive the nonlinear evolution of the parallel momentum density per ion mass, i.e., $nU_\parallel \equiv 2\pi\int d\mu dv_\parallel B^*Fv_\parallel$, by taking a moment of the nonlinear gyrokinetic equation (3), or equivalently of a conservative form of the nonlinear gyrokinetic equation [Eq. (24) of Ref. 28]:

$$\frac{\partial(FB^*)}{\partial t} + \nabla\cdot\left(FB^*\frac{d\mathbf{R}}{dt}\right) + \frac{\partial}{\partial v_\parallel}\left(FB^*\frac{dv_\parallel}{dt}\right) = 0. \tag{6}$$

With the Mach number using the sound speed $\mathrm{M}_s \equiv U_0/C_s$, we adopt an ordering $k_\theta\rho_s > (a/qR)\mathrm{M}_s$, and assume $\mathrm{M}_s < 1$, so that we can ignore $\mathbf{B}\cdot\nabla nU_\parallel^2$ in comparison to $\mathbf{B}\cdot\nabla P_\parallel$. The pressure moments per unit mass are defined as usual.[24] With these considerations, we can write a nonlinear evolution equation for the parallel momentum, by multiplying Eq. (6) by $v_\parallel$ and integrating over the velocity space, to obtain

$$\frac{\partial}{\partial t}(m_i n U_\parallel) = -c\mathbf{b}\times\nabla\delta\phi\cdot\nabla\left(\frac{m_i n U_\parallel}{B}\right) - 2m_i n U_\parallel \mathbf{b} \times(\mathbf{b}\cdot\nabla)\mathbf{b}\cdot\nabla\delta\phi - \frac{m_i^2 c}{e_i}\mathbf{b}\times\nabla B\cdot\nabla\left(\frac{P_\perp U_\parallel}{B^2}\right) - 3\frac{m_i^2 c}{e_i}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}\cdot\nabla\left(\frac{P_\parallel U_\parallel}{B}\right) - n_i e_i \mathbf{b}\cdot\nabla\delta\phi - \mathbf{b}\cdot\nabla P_\parallel. \tag{7}$$

The second term on the right-hand side (RHS) of Eq. (7) originates from the magnetic curvature modification of the parallel acceleration in Eq. (5). The last two terms are the origin of the $\mathbf{E}\times\mathbf{B}$ shear induced residual stress.[22] The $\mathbf{E}\times\mathbf{B}$ shear has been known to produce a nondiffusive radial flux of the parallel flow in simple geometry.[18] The physics of residual stress has been extensively discussed in Ref. 22. Therefore, from this point, we do not keep these terms in this paper, which focuses only on the inward pinch driven by toroidal effects. In low-β plasmas, $\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}=(\nabla\times\mathbf{b})_\perp \simeq -\mathbf{B}\times\nabla(1/B)$. With this approximation, by combining the second term on the RHS into the first term, Eq. (7) can be further simplified to

$$\frac{\partial}{\partial t}\left(\frac{m_i n U_\parallel}{B^3}\right) = -\frac{c\mathbf{b}\times\nabla\delta\phi}{B}\cdot\nabla\left(\frac{m_i n U_\parallel}{B^3}\right) - \frac{m_i^2 c\mathbf{b}\times\nabla B}{e_i B^3}\cdot\nabla\left(\frac{P_\perp U_\parallel}{B^2}\right) - 3\frac{m_i^2 c\mathbf{b}\times\nabla B}{e_i B^4}\cdot\nabla\left(\frac{P_\parallel U_\parallel}{B}\right). \tag{8}$$

It is also instructive to write Eq. (8) in a continuity form, anticipating that we will eventually calculate the radial flux of the parallel angular momentum, and that the divergence of that term will determine the time evolution of the mean angular momentum density. The algebra is nontrivial due to the fact that the $\mathbf{E}\times\mathbf{B}$ flow is no longer incompressible in an inhomogeneous plasma [i.e., $\nabla\cdot\mathbf{u}_E\equiv\nabla\cdot(c\mathbf{b}\times\nabla\phi/B)\neq 0$]. Fortunately, we can make a low-β approximation, i.e., $\nabla\cdot(\mathbf{u}_E B^2)=4\pi\mathbf{J}\cdot\nabla\phi\ll B^2\nabla\cdot\mathbf{u}_E$, to make further analytic progress. We can then rewrite Eq. (8) as follows:

$$\frac{\partial}{\partial t}\left(\frac{m_i n U_\parallel}{B}\right) = -\nabla\cdot\left(\frac{m_i n U_\parallel}{B}\mathbf{u}_E\right) - \frac{m_i^2 c}{e_i}\nabla \times\left[(\mathbf{b}\times\nabla B)\left(\frac{T_\perp n U_\parallel}{B^3}\right)\right] - \frac{m_i^2 c}{e_i}\nabla\cdot\left[(3\mathbf{b}\times\nabla B)\left(\frac{T_\parallel n U_\parallel}{B^3}\right)\right]. \tag{9}$$

It is important to recognize the following facts. First, since $B\propto 1/R$ in tokamaks, we note that $m_i n U_\parallel/B \propto m_i n U_\parallel R = m_i n R^2\omega_\parallel$ is the parallel angular momentum in tokamak geometry, with $\omega_\parallel$ being the parallel angular rotation frequency, and $I\equiv m_i n R^2$ being the density of the moment of inertia. The expression "$\nabla\cdot(m_i\delta(nU_\parallel/B)\delta\mathbf{u}_E)$" essentially leads to the radial flux of the parallel angular momentum. We note that the relation $B\propto 1/R$ does not hold for all geometries. For instance, in a spherical torus, further refinement in the analysis using a more realistic MHD equilibrium is desirable. The rest of the terms in Eq. (9) can be identified as the geodesic curvature driven momentum flux $\mathbf{\Pi}_{\mathrm{Geo}}$,[24] which is subdominant to the standard $\mathbf{E}\times\mathbf{B}$ drift induced contribution from the first term on the RHS of Eq. (9). So, as far as the evolution of the mean angular momentum profile is concerned, we will ignore these terms from now on.

Typically, transport analyses[33] deal with the temporal evolution of the flux-surface-averaged toroidal angular momentum density $\langle m_i n R^2\rangle\omega_\phi$, where the toroidal angular frequency is a flux function. In this paper, we use a set of variables (ψ,θ,ζ) to denote the radial, poloidal, and toroidal coordinates, respectively. The equilibrium magnetic field $\mathbf{B}$ is given by

$$\mathbf{B}=\nabla\zeta\times\nabla\psi+g(\psi)\nabla\zeta, \tag{10}$$

where $d\psi=RB_\theta dr$, and the toroidal magnetic field strength is given by $B_\phi=g(\psi)/R$. Following the same procedure described in Ref. 24, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}(\langle m_i n R^2\rangle\omega_\parallel) &= -\langle\nabla\cdot\mathbf{\Pi}_{\mathrm{Ang}}\rangle \\ &= -\frac{1}{V'}\frac{\partial}{\partial\psi}[V'\langle\mathbf{\Pi}_{\mathrm{Ang}}\cdot\nabla\psi\rangle] \\ &= -\frac{1}{V'}\frac{\partial}{\partial\psi} \times\left[V'\left\langle m_i\delta(nU_\parallel R)\frac{c}{B}\mathbf{b}\times\nabla\delta\phi^*\cdot\nabla\psi\right\rangle\right] \\ &\simeq -\frac{1}{V'}\frac{\partial}{\partial\psi}\left[V'\left\langle m_i c R\sum_{\mathbf{k}}\delta(nU_\parallel R)_{\mathbf{k}}\frac{\partial}{\partial\zeta}\delta\phi_{\mathbf{k}}^*\right\rangle\right].\end{aligned} \tag{11}$$

Here, we used the fact that $k_\parallel\ll k_\perp$. For the evaluation of the nonlinear turbulent flux of angular momentum $\mathbf{\Pi}_{\mathrm{Ang}}^{\mathrm{Turb}}$ in Eq. (9), the expression for the perturbed angular momentum $\delta(nU_\parallel R)$ can be obtained from Eq. (7). In $\mathbf{k}$-space, it can be written as

$$\begin{aligned}&[-i\omega_{\mathbf{k}}+\Delta\omega_{\mathbf{k}}+i(3\omega_{d\parallel\mathbf{k}}+\omega_{d\perp\mathbf{k}})]\delta(nU_\parallel R)_{\mathbf{k}} \\ &= -\delta v_{r\mathbf{k}}\hat{\mathbf{e}}_\psi\cdot\nabla(n_0U_0R) - i2\omega_{d\parallel\mathbf{k}}\frac{e\delta\phi_{\mathbf{k}}}{T_\parallel}n_0U_0R \\ &\quad - i\left(3\omega_{d\parallel\mathbf{k}}\frac{\delta T_{\parallel\mathbf{k}}}{T_\parallel}+\omega_{d\perp\mathbf{k}}\frac{\delta T_{\perp\mathbf{k}}}{T_\perp}\right)n_0U_0R.\end{aligned} \tag{12}$$

It is noteworthy that $\delta(nU_\parallel R)$ can be driven not only by the radial gradient of $nU_\parallel R$, which eventually leads to a diffusive radial flux, but also by the gradient of B. This is contained in the definitions of $\omega_{d\mathbf{k}}$. $\omega_{d\parallel\mathbf{k}}\equiv(cT_\parallel/eB)\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}\cdot\mathbf{k}$ is the curvature drift of thermal ions, while $\omega_{d\perp\mathbf{k}}\equiv(cT_\perp/eB^2)\mathbf{b}\times\nabla B\cdot\mathbf{k}$ is the grad-B drift of thermal ions. This leads to a non-diffusive radial flux of the parallel momentum, as long as electrostatic fluctuations are present. The second term on the RHS has been identified as the "turbulent equipartition (TEP) pinch" in Ref. 24, based on its insensitivity to details such as the dispersion relation of ambient fluctuations. This TEP part of the TurCo pinch is the main subject of this paper.

On the other hand, the third term on the RHS is related to ion temperature fluctuations whose magnitude and phase relationship with respect to $\delta\phi$ depend on the nature of the fluctuations (for instance, depending on whether it is ITG or TEM dominated). We have classified this part as the curvature driven thermoelectric (CTh) pinch in Ref. 24, since both ion thermal effects and magnetic curvature are required for this term. Due to its mode dependency, we cannot make any further generic statement on this part of the TurCo pinch, except that it is expected to be smaller than the TEP pinch in the hot electron mode regime ($T_e > T_i$) as expected in OH and electron heated plasmas.[6,8] The expression multiplying $\delta(nU_\parallel R)_{\mathbf{k}}$ on the left-hand side of Eq. (12) is the $(\mathbf{k},\omega)$-space version of the renormalized propagator, in which $\Delta\omega_{\mathbf{k}}$ is the decorrelation rate that originates from the $\mathbf{E}\times\mathbf{B}$ nonlinear term in Eq. (8). Here, $\boldsymbol{\tau}_{c\mathbf{k}} \equiv [-i\omega_{\mathbf{k}} + \Delta\omega_{\mathbf{k}} + i(3\omega_{d\parallel\mathbf{k}} + \omega_{d\perp\mathbf{k}})]^{-1}$ is the inverse of the propagator. Its real part, which is positive definite and independent of mode propagation direction, corresponds to the correlation time of the turbulence.

Now, we can explicitly evaluate the diffusive part and the TEP part of the TurCo pinch of angular momentum flux and can calculate its divergence from Eq. (11). While one can measure the angular momentum density flux directly from nonlinear turbulence simulations, transport analysis[33] of experimental data involves flux-surface-averaged quantities. We denote the flux-surface-average by $\langle\ldots\rangle$. From the first term on the RHS of Eq. (12), we obtain the usual diffusive part of the radial component of the parallel angular momentum density flux:

$$\begin{aligned}\langle \mathbf{\Pi}_{\mathrm{Ang}}^{\mathrm{Diff}} \cdot \nabla\psi\rangle &= -\left\langle \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} |\delta v_{r\mathbf{k}}|^2\, \nabla (m_i n_0 U_0 R)\cdot\nabla\psi \right\rangle \\ &= -\chi_{\mathrm{Ang}} \left\langle (RB_\theta)^2 \frac{\partial}{\partial\psi}(m_i n_0 R^2 \omega_\parallel)\right\rangle. \end{aligned} \tag{13}$$

Here, the flux-surface-averaged "angular momentum density diffusivity" can be defined as

$$\begin{aligned}\chi_{\mathrm{Ang}} &\equiv \left\langle \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} |\delta v_{r\mathbf{k}}|^2 \right\rangle \\ &= \left\langle \left(\frac{c}{RB_\theta}\right)^2 \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} \ell^2 |\delta\phi_{\mathbf{k}}|^2 \right\rangle. \end{aligned} \tag{14}$$

To obtain Eq. (14), we used the following identities: $|\nabla\psi| = RB_\theta$, $\mathbf{b}\times\hat{\mathbf{e}}_\psi\cdot\mathbf{k} = \ell B/RB_\theta$, and $\delta v_{r\mathbf{k}} = -i(c\ell/RB_\theta)\delta\phi_{\mathbf{k}}$ with ℓ=toroidal mode number. From the second term on the RHS of Eq. (12), we obtain the turbulent equipartition (TEP) part of the toroidal angular momentum density TurCo pinch; i.e.,

$$\langle \mathbf{\Pi}_{\mathrm{Ang}}^{\mathrm{TEP}} \cdot \nabla\psi\rangle = -2\left\langle \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} \delta v_{r\mathbf{k}}^{*} i\left(\omega_{d\parallel\mathbf{k}} \frac{e\,\delta\phi_{\mathbf{k}}}{T_\parallel}\right) m_i n_0 R^2 \omega_\parallel R B_\theta \right\rangle = \langle m_i n_0 R^3 B_\theta\rangle \omega_\parallel V_{\mathrm{Ang}}^{\mathrm{TEP}}. \tag{15}$$

Here, the flux-surface-averaged "TEP angular momentum pinch" can be defined as

$$\begin{aligned} V_{\mathrm{Ang}}^{\mathrm{TEP}} &\equiv -2\left\langle \sum_{\mathbf{k}} i\ \mathrm{Re}\ \tau_{c\mathbf{k}} \delta v_{r\mathbf{k}}^{*} \omega_{d\parallel\mathbf{k}} \frac{e\,\delta\phi_{\mathbf{k}}}{T_\parallel} \right\rangle \\ &= 2\left\langle \frac{c}{RB_\theta} \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} \ell\, \omega_{d\parallel\mathbf{k}} \frac{e}{T_\parallel} |\delta\phi_{\mathbf{k}}|^2 \right\rangle. \end{aligned} \tag{16}$$

Using the identity $\omega_{d\parallel,\mathbf{k}}(0) = -(cT_\parallel/e_i RB_\theta)\ell/R$ at the low-B side midplane (θ=0), we can write

$$\begin{aligned} V_{\mathrm{Ang}}^{\mathrm{TEP}} &= -2\left\langle \frac{1}{R}\left(\frac{c}{RB_\theta}\right)^2 \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} \ell^2 \frac{\omega_{d\parallel\mathbf{k}}(\theta)}{\omega_{d\parallel\mathbf{k}}(0)} |\delta\phi_{\mathbf{k}}|^2 \right\rangle \\ &= -2\left\langle \frac{1}{R} \sum_{\mathbf{k}} \mathrm{Re}\ \tau_{c\mathbf{k}} \frac{\omega_{d\parallel\mathbf{k}}(\theta)}{\omega_{d\parallel\mathbf{k}}(0)} |\delta v_{r,\mathbf{k}}|^2 \right\rangle. \end{aligned} \tag{17}$$

Note that the details of the turbulence dynamics do *not* enter the expression for the TEP pinch, which is insensitive to the mode propagation direction, etc.; it depends *only* upon the correlation time and the spectrum of radial $\mathbf{E}\times\mathbf{B}$ velocities. From Eqs. (14) and (17), we obtain

$$V_{\mathrm{Ang}}^{\mathrm{TEP}} \simeq -\frac{2F_{\mathrm{balloon}}}{R_0}\chi_{\mathrm{Ang}}, \tag{18}$$

with $F_{\mathrm{balloon}} \equiv \langle \omega_{d\mathbf{k}}(\theta)|\delta\phi(\theta)|^2\rangle / \langle \omega_{d\mathbf{k}}(0)|\delta\phi(\theta)|^2\rangle$. Note that, for the TEP pinch of the (linear) momentum density $nU_\parallel$, an additional contribution from $\omega_{d\perp\mathbf{k}}$ appears in the TEP pinch expression due to the fact that $R \propto 1/B$ [see Eq. (14) and Eq. (37) of Ref. 24]. Therefore, we have

$$V_{\mathrm{Mom}}^{\mathrm{TEP}} \simeq -\frac{3F_{\mathrm{balloon}}}{R_0}\chi_{\mathrm{Mom}}. \tag{19}$$

III. TURBULENT EQUIPARTITION PINCH OF PARALLEL ANGULAR MOMENTUM

After the quasilinear derivation from the gyrokinetic equation and the turbulent equipartition (TEP) interpretation of the mode-independent part of the turbulent convective (TurCo) pinch of angular momentum density was published in Ref. 24, we presented a simpler and more intuitive derivation of the TEP pinch based on an *ansatz* of local angular momentum conservation and homogenization.[29] In this section, we recapitulate the essence of the two different approaches and clarify the relation between them, and thereby put the TEP interpretation of the mode-independent part of the TurCo pinch, which was originally derived from the gyrokinetic equation, on a firmer theoretical ground.

As discussed in relation to Eq. (9) in Sec. II, if we ignore the parallel dynamics and the flux of momentum due to the thermal (velocity-dependent) magnetic drift, we can write the evolution of the angular momentum density as

$$\frac{\partial}{\partial t}(m_i n U_\parallel R) + \nabla \cdot (m_i n U_\parallel R \mathbf{u}_E) = 0, \tag{20}$$

noting that $B \propto 1/R$ in tokamaks. Here, the unique role played by the angular momentum density in toroidal geometry should be appreciated. For instance, it is not possible to construct a simple continuity equation for the linear momentum density $m_i n U_\parallel$ in toroidal geometry, as is obvious from Eq. (7), even in the absence of ion thermal effects.

Indeed, Eq. (20) is the starting point of our TEP interpretation,[24] and of a simple physical derivation thereof, as presented in Ref. 29. As is familiar from the TEP theory of the particle pinch[34–37] and of the angular momentum pinch,[24,29] the quantity that gets homogenized (mixed) by turbulence is the one that is *locally* conserved, i.e., a scalar field A, which satisfies the relation

$$\frac{d}{dt}A = \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)A = \nu \nabla^2 A, \tag{21}$$

where $\nu \nabla^2 A$ is the diffusive dissipation on small scales. In toroidal geometry, the $\mathbf{E} \times \mathbf{B}$ flow is compressible due to the inhomogeneous magnetic field, and as a consequence, the angular momentum density $m_i n U_\parallel R$ cannot satisfy a relation such as Eq. (21). For a low-β tokamak equilibrium, we have shown that $\nabla \cdot (\mathbf{u}_E B^2) \ll B^2 \nabla \cdot \mathbf{u}_E$. Therefore, considering $\mathbf{u}_E B^2$ as *incompressible*, we can write

$$\frac{d}{dt}\left(\frac{m_i n U_\parallel R}{B^2}\right) = \left(\frac{\partial}{\partial t} + \mathbf{u}_E \cdot \nabla\right)\frac{m_i n U_\parallel}{B^2} R = 0 \tag{22}$$

up to the diffusive dissipation on small scales. This is the *local* conservation of the magnetically weighted angular momentum (MWA) density[24] which is the central element of the TEP TurCo pinch of angular momentum. This relation can be also obtained from Eq. (8), by ignoring the ion thermal effects that eventually lead to the CTh part of the TurCo pinch.

According to the homogenization theory,[38] a scalar field that is locally advected by a shearing flow, within a closed streamline in the presence of diffusion (i.e., the MWA, $m_i n U_\parallel R/B^2$ in this case) will eventually be mixed or homogenized. It is expected that turbulence driven sheared $\mathbf{E} \times \mathbf{B}$ zonal flows[39,40] coexist with the ambient turbulence in OH and L-mode plasmas. This will tend to speed up the process of homogenization within the same flux surface via the random shearing.[41,42]

Note that we can rewrite Eq. (22) as

$$\frac{\partial}{\partial t} m_i n U_\parallel R + B^2 \mathbf{u}_E \cdot \nabla\left(\frac{m_i n U_\parallel R}{B^2}\right) = 0 \tag{23}$$

and regard $B^2 \mathbf{u}_E$ as an *incompressible flow*. This homogenization by turbulent incompressible flow occurs via diffusion of a locally conserved quantity; i.e., the MWA density. For the transport of angular momentum in which we are interested, in the context of magnetic confinement physics, this diffusion of MWA density manifests itself as a combination of the TEP pinch and the diffusion of the angular momentum density. Thus, the homogenization tends towards a state where $\nabla(m_i n U_\parallel R/B^2) \to 0$, linking $\nabla U_\parallel$ to ∇B. This is equivalent to an off-diagonal, inward pinch.

Indeed, the physical origin of the ∇B-driven piece of the TurCo momentum pinch is easily revealed by considering the radial quasilinear turbulent flux of the magnetically weighted angular momentum (MWA) density. While the detailed derivation can be found in Ref. 24, it is crucial to note that the total flux of the parallel angular momentum density $n U_\parallel R$ consists of (i) a diffusive piece, driven by $\nabla(n U_\parallel R)$ and (ii) an off-diagonal, or convective piece, driven by $\nabla(1/B^2)$. Since $\nabla(1/B^2) \cdot \nabla \psi > 0$, on the low-$B$ side, where the fluctuation amplitude peaks, this piece is indeed a pinch, and produces an inward flux of parallel angular momentum density. The pinch term described above corresponds to the ∇B-driven TEP component of the TurCo flux of angular momentum, since it is *not* driven by a thermodynamic force, such as ∇T_i or ∇n.

Some comments comparing the TEP theories for angular momentum and density are appropriate here. Pinches in both quantities originate from the local advection and homogenization (mixing) of the locally conserved quantities. These quantities are *magnetically weighted*, due to the fact that the $\mathbf{E} \times \mathbf{B}$ flow in an inhomogeneous $\mathbf{B}$ field is no longer incompressible. These are $n U_\parallel R/B^2$ in the case of angular momentum transport, and n/B in the case of a simple density transport model.[36] On the surface, a relation such as $(d/dt) \times (n/B) = 0$ reminds one of the "frozen-in law." Indeed, one can interpret the TEP theory of angular momentum density as a consequence of the frozen-in law (linking n and B), and the fact that angular momentum density and density obey the same continuity equation involving the $\mathbf{E} \times \mathbf{B}$ flow, approximately (linking $n U_\parallel R$ and n).[29] This interpretation illustrates the similarity between the particle pinch and the angular momentum density pinch explicitly.

However, great care should be exercised in using the ion density continuity equation alone (without considering the electron dynamics and quasineutrality), in studying particle transport and its effect on momentum transport. Actually, other magnetic fusion relevant TEP theories for density involve magnetically trapped electrons.[34,35,37] The dynamics for these is governed by bounce-kinetics in which parallel streaming averages out, and thus is constrained by conservation of *two* adiabatic invariants; namely, the magnetic moment μ and the second invariant J, the bounce action. In contrast, our momentum pinch theory does not require the conservation of J. While their explicit formulas are different, due to the trapped particle effects, for instance, both these theories yield pinches with roughly comparable magnitudes when normalized to the corresponding diffusivities (χ_{Ang} and D_{ptl}), respectively. The TEP theories for angular momentum and density are summarized in Table I.

As stated in Ref. 24, we have chosen to formulate the problem in terms of the angular momenum density, rather

TABLE I. Turbulent equipartition pinches of particles and angular momentum.

Quantity of interest in transport problem	Density n (Refs. 34–37)	Angular momentum density $nU_{\parallel}R$ or parallel momentum density $nU_{\parallel}$
Locally conserved quantity that gets homogenized	n/B in two-dimensional slab (Ref. 36)	Magnetically weighted momentum density $nU_{\parallel}R/B^2$ in torus (Ref. 24)
Inward pinch velocity of transported quantity	$V_{\text{pinch}}/D \simeq -[(1/2)+(4\hat{s}/3)]/R_0$ (Ref. 37)	$V_{\text{Ang}}^{TEP}/\chi_{\text{Ang}} \simeq -2/R_0$ $V_{\text{Mom}}^{TEP}/\chi_{\text{Mom}} \simeq -3/R_0$
Possible relevance to experiments	L-mode plasmas in various tokamaks (Refs. 43–46)	Comparisons in progress NSTX (Ref. 15), JET (Ref. 47)

than the flow $U_{\parallel}$, because it is the most natural quantity for a theoretical formulation that identifies some generic features from the thermodynamic point of view, but does not explicitly specify the density dynamics. Indeed, neoclassical theories of momentum transport are also formulated in terms of the angular momentum.[31,48,49] We also note that this is the quantity that gets perturbed directly in transient momentum transport analysis using NBI.[14] Of course, it is the flow that gets measured in experiments, and the effects of particle transport should be taken into account in theory-experiment comparisons, unless the particle transport is negligible, as is often claimed to be the case in the cores of tokamaks. A careful treatment of particle transport for the nonadiabatic electron response, and of its coupling to momentum transport, is one of the outstanding issues that should be addressed in the future.

IV. GYROKINETIC FORMULATION IN THE ROTATING FRAME

In this section, we discuss theoretical issues that arise when one calculates the turbulence driven radial flux of parallel flow in the rotating frame. In particular, we identify terms in the gyrokinetic equation that lead to the diffusive flux and the momentum pinch, respectively. In the laboratory frame, the advection of the mean parallel flow via the fluctuating $\mathbf{E}\times\mathbf{B}$ drift, which will eventually lead to the diffusive flux of parallel flow, is described by a term $(c/B)\mathbf{b}\times\nabla\langle\langle\delta\phi\rangle\rangle\cdot\nabla F_0$, where F_0 contains the radially dependent mean flow U_0 explicitly. In most cases, it is taken as a shifted Maxwellian distribution function $F_0 \propto \exp[-(v_{\parallel}-U_0)^2/2v_{Ti}^2]$, where $v_{\parallel}$ is one of the independent variables. On the other hand, in the rotating frame of reference, $c_{\parallel}\equiv v_{\parallel}-U_0$, rather than $v_{\parallel}$, is an independent variable. Therefore, with $F_0 \propto \exp[-c_{\parallel}^2/2v_{Ti}^2]$, the $(c/B)\mathbf{b}\times\nabla\langle\langle\delta\phi\rangle\rangle\cdot\nabla F_0$ term contains only the advection of the mean temperature gradient and the mean density gradient, but *not* the parallel flow gradient. We note that the ∇B-driven term is also contained in this term if μ rather than $v_{\perp}$ is used as an independent variable. It is not widely known which term is responsible for the diffusive flux of parallel flow in a formulation in the rotating frame. We demonstrate that the magnetic curvature modification of the parallel acceleration in the nonlinear gyrokinetic equation in the laboratory frame,[28] which was shown to be responsible for the TEP part of the TurCo pinch of angular momentum density in our previous work,[24] is closely related not only to the term responsible for the diffusive flux in the rotating frame, but also to the Coriolis drift coupling to the perturbed electric field. The Coriolis force, which is familiar in the geophysical fluid dynamics context, for instance,[50] also appears in the drift kinetic[31] and gyrokinetic[32] formulations in the rotating frame, as it should. In Ref. 23, it was shown that the Coriolis drift can lead to a momentum pinch in toroidal plasmas. By illustrating this intriguing manifestation of physics related to momentum transport in the rotating reference frame, we elucidate some crucial points that were not presented in Ref. 23.

The nonlinear toroidal gyrokinetic equation with proper conservation laws[28] is presented in Eqs. (3)–(5). In the reference frame moving with $\mathbf{U}_0$, it can be written in terms of $(\mu, c_{\parallel}, \mathbf{R})$, where $c_{\parallel}$ is the parallel component of the relative velocity,[32]

$$\frac{\partial F}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\nabla F + \frac{dc_{\parallel}}{dt}\frac{\partial F}{\partial c_{\parallel}} = 0, \tag{24}$$

with

$$\frac{d\mathbf{R}}{dt} = \mathbf{U}_0 + c_{\parallel}\mathbf{b} + \frac{c\mathbf{b}}{e_iB^*} \times[e_i\nabla\langle\langle\delta\phi\rangle\rangle + m_i\mu\nabla B + m_i\mathbf{U}_0^*\cdot\nabla\mathbf{U}_0^*] \tag{25}$$

and

$$\frac{dc_{\parallel}}{dt} = -\frac{\mathbf{B}^*}{m_iB^*}\cdot[e_i\nabla\langle\langle\delta\phi\rangle\rangle + m_i\mu\nabla B + m_i\mathbf{U}_0^*\cdot\nabla\mathbf{U}_0^*]. \tag{26}$$

Here, the gyrokinetic Vlasov equation (24) is written in terms of the guiding center distribution function $F(\mathbf{R},\mu,c_{\parallel},t)$, with $\mu\equiv v_{\perp}^2/2B$. $\mathbf{B}^*$ is defined by

$$\mathbf{B}^* \equiv \mathbf{B} + \frac{m_ic}{e_i}\nabla\times(c_{\parallel}\mathbf{b}+\mathbf{U}_0).$$

The δf version of the gyrokinetic equation is

$$\frac{\partial\delta f}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\nabla\delta f + \frac{dc_{\parallel}}{dt}\frac{\partial\delta f}{\partial c_{\parallel}} = -\frac{d\mathbf{R}^{(1)}}{dt}\cdot\nabla F_0 - \frac{dc_{\parallel}^{(1)}}{dt}\frac{\partial F_0}{\partial c_{\parallel}}, \tag{27}$$

with

$$\frac{d\mathbf{R}^{(1)}}{dt} = \frac{c\mathbf{b}}{B^*} \times \nabla\langle\langle \delta\phi \rangle\rangle$$

and

$$\frac{dc_\parallel^{(1)}}{dt} = -\frac{e_i}{m_i}\frac{\mathbf{B}^*}{B^*} \cdot \nabla\langle\langle \delta\phi \rangle\rangle.$$

Here, $\mathbf{U}_0^* \equiv \mathbf{U}_0 + c_\parallel \mathbf{b}$, and we take $\mathbf{U}_0 = R^2\omega_\phi(\psi)\nabla\zeta = R\omega_\phi(\psi)\mathbf{e}_\zeta$. Therefore,

$$\nabla \times \mathbf{U}_0^* = c_\parallel \nabla \times \mathbf{b} + 2R\omega_\phi(\psi)\nabla R \times \nabla\zeta + R^2 \nabla \omega_\phi(\psi) \times \nabla\zeta. \quad (28)$$

While ω_ϕ, rather than $U_0 = R\omega_\phi$, is a flux function in most cases, it is more customary to use U_0 and ∇U_0 as the main variables in momentum transport analysis.[33] For this purpose, one can write Eq. (28) as

$$\nabla \times \mathbf{U}_0^* = c_\parallel \nabla \times \mathbf{b} + U_0 \nabla R \times \nabla\zeta + R \nabla U_0 \times \nabla\zeta. \quad (29)$$

The acceleration due to the perturbed electric field in the rotating frame can then be written as

$$\frac{dc_\parallel^{(1)}}{dt} = -\frac{e_i}{m_i B^*}\mathbf{B} \cdot \nabla\langle\langle \delta\phi \rangle\rangle - \frac{c}{B^*}\{c_\parallel \nabla \times \mathbf{b} + U_0 \nabla R \times \nabla\zeta + R \nabla U_0 \times \nabla\zeta\} \cdot \nabla\langle\langle \delta\phi \rangle\rangle. \quad (30)$$

Now, after making the approximation $\mathbf{U}_0 \simeq U_0\mathbf{b}$, it is clear that the $\mathbf{E}\times\mathbf{B}$ advection of the mean parallel flow can be described by the contribution of the last term in Eq. (30) to the last term of Eq. (27), i.e.,

$$\frac{c}{B^*} \nabla U_0 \times \mathbf{b} \cdot \nabla\langle\langle \delta\phi \rangle\rangle \frac{\partial F_0}{\partial c_\parallel} \simeq \frac{c}{B}\mathbf{b} \times \nabla\langle\langle \delta\phi \rangle\rangle \cdot \nabla U_0 \frac{\partial F_0}{\partial c_\parallel}, \quad (31)$$

with the approximation $B^* \simeq B$, which should be safe, away from the separatrix where the magnetic shear diverges.[51] Indeed, this expression is identical to a part coming from the radial gradient of the mean parallel flow in $(c/B)\mathbf{b} \times \nabla\langle\langle \delta\phi \rangle\rangle \cdot \nabla F_M$ from the formulation in the laboratory frame with $F_M \propto \exp[-(v_\parallel - U_0)^2/2v_{Ti}^2]$.[24] Accordingly, this term will eventually lead to the diffusive flux of the parallel flow if one performs a standard quasilinear calculation[24] consistently. We also note that this term is responsible for destabilization of the parallel shear flow instability,[2,52] as noted in the context of the gyrokinetic formulation in slab geometry in the moving frame; i.e., Eq. (2) of Ref. 53. The third term on the right-hand side of Eq. (30) is the Coriolis drift coupling to the perturbed electric field, which leads to a part of the inward pinch of parallel flow discussed in Ref. 23. It is useful to note from Eq. (2) of Ref. 53 that the parallel flow gradient term (the last on the RHS) remains, while other toroidal geometry-related terms (the second term on the RHS related to the magnetic curvature, and the third term related to the Coriolis drift) in Eq. (30) of the present paper disappear in slab or cylinder geometry.

The derivation in Ref. 23 contains some ambiguities. The term proportional to $\nabla\omega_\phi$ [the last term of Eq. (28)] is absent in the gyrokinetic equation and the associated guiding center equations of motion in Ref. 23. Probably this is what is meant by the assumption of *a constant rigid body toroidal rotation*. However, the parallel flow gradient term appears in the fluid moment equation without an explanation of its origin in the gyrokinetic equation from which the fluid moments are taken. It appears in a form equivalent to the last term of Eq. (29), rather than in the form of the last term of Eq. (28), which has been dropped originally. As a consequence, the second term of Eq. (29) has been double-counted. We believe that this leads to an overestimation of the Coriolis drift induced inward pinch of parallel flow in Ref. 23. The remaining part of the parallel flow pinch in toroidal geometry can originate from the second term on the left-hand side of Eq. (27).

V. SCALING OF MOMENTUM PINCH

The scaling and magnitude of $V_{\mathrm{pinch}}/\chi_\phi$ are of great practical interest since this determines the overall peakedness of rotation profiles in the region where external torque input and residual stress driven by the $\mathbf{E}\times\mathbf{B}$ shear are absent. Therefore, careful theoretical underpinning of scalings indicated by various theories is necessary to make comparisons to experiments and simulations more meaningful. Furthermore, any extrapolation to larger future machines, such as ITER, can be considered credible only after a proper understanding of the theory and its validity regimes.

The TEP part of the TurCo pinch[24] is a common element of the turbulence driven inward pinch in toroidal geometry, which is independent of the details of the ambient turbulence as long as it has a significant electrostatic component, and its perpendicular correlation length is larger than, or comparable to, ion gyroradii. In the absence of particle flux, the predicted TEP pinch velocity satisfies $V_{\mathrm{Ang}}^{\mathrm{TEP}}/\chi_{\mathrm{Ang}} \simeq -2/R_0$ for angular momentum $U_\parallel R$, and $V_{\mathrm{Mom}}^{\mathrm{TEP}}/\chi_{\mathrm{Mom}} \simeq -3/R_0$ for parallel flow $U_\parallel$. Its origin is the magnetic curvature $\nabla\times\mathbf{b}$, which exists in toroidal experiments. Since this makes the $\mathbf{E}\times\mathbf{B}$ flow compressible, the magnetically weighted angular momentum density $nU_\parallel R/B^2$ (a locally conserved quantity, approximately), rather than the angular momentum density $nU_\parallel R$, gets homogenized (mixed) by turbulence. The inward pinch of the "observed" quantity $nU_\parallel R$ is a manifestation of this tendency towards homogenization or equipartition in the space of motion invariants. The scaling with respect to R, i.e., $V_{\mathrm{pinch}}/\chi_\phi \to 0$ as $1/R \to 0$, is consistent with this physical interpretation based on the geometric effect $B \propto 1/R$.

While this TEP part of the TurCo pinch is the common element of the turbulence driven inward pinch, there exist other physical mechanisms that can possibly lead to a stronger inward pinch depending on plasma parameters and the nature of the ambient turbulence. The CTh (curvature driven thermoelectric) part of the TurCo pinch depends on δT_i and its phase relationship with respect to $\delta\phi$. While we did not pursue a detailed analytic prediction in our previous paper[24] due to its algebraic complexity, this CTh part of the TurCo pinch should also have the property that $V_{\mathrm{pinch}}/\chi_\phi \to 0$ as $1/R \to 0$, since it is also related to the magnetic curvature. Note that due to the hybrid nature of toroidal instabilities[54] yielding $\omega \propto (\omega_{*\mathrm{pi}}\omega_{Di})^{1/2}$, a scaling such as $V_{\mathrm{pinch}}^{\mathrm{CTh}}/\chi_\phi \propto -1/(RL_p)^{1/2}$ is not impossible. Another mechanism that is not captured by the TEP theory of the momentum pinch is the wave-particle resonant interaction. The importance of this mechanism in momentum transport has been recognized with varying degrees of theoretical generality.[17,19,20,25] Since this mechanism must exist in simple geometry (in the absence of magnetic curvature and toroidicity), a "scaling" such as $V_{\mathrm{pinch}}/\chi_\phi \propto -1/L_\perp$ is possible, with $L_\perp$ from the radial gradient in either temperature or density, while toroidal effects can modify the coefficient in front.

Reference 23 presents a simple analytic formula for the inward pinch of parallel flow, for pure ITG instability based on fluid moment equations. It has been attributed to the Coriolis drift effect. Its scaling with respect to the density gradient length, i.e., $V_{\mathrm{pinch}}/\chi_\phi = -1/L_n - 4/R$, is puzzling from a theoretical point of view. As Eq. (2) of Ref. 53 suggests, any toroidal effect (including that of the Coriolis drift) should vanish in simple (slab or cylinder) geometry, i.e., in the limit $R \to \infty$. However, the density gradient driven inward pinch persists in this limit according to Ref. 23. If such a linear dependence on L_n^{-1} is real, it should come from a physics mechanism which exists in simple geometry, such as a wave-particle resonant interaction.[25]

VI. CONCLUSIONS

In this paper, we put the TEP interpretation of the mode-independent part of the TurCo pinch,[24] which was originally derived from the gyrokinetic equation, on a firmer and more transparent theoretical ground. The principal results of this paper are:

(i) The essence of a quasilinear derivation from the gyrokinetic equation has been recapitulated, and its relation to a simpler and more intuitive derivation based on an *ansatz* of local angular momentum conservation and homogenization[29] has been elucidated.

(ii) Our quantitative predictions on the pinch velocities are $V_{\mathrm{Ang}}^{\mathrm{TEP}} \simeq -(2/R_0)\chi_{\mathrm{Ang}}$, for the angular momentum density $nU_\parallel R$, and $V_{\mathrm{Mom}}^{\mathrm{TEP}} \simeq -(3/R_0)\chi_{\mathrm{Mom}}$, for the parallel momentum density $nU_\parallel$.

(iii) We have demonstrated that the magnetic curvature modification of the parallel acceleration in the nonlinear gyrokinetic equation in the laboratory frame,[28] which was shown to be responsible for the TEP part of the TurCo pinch of angular momentum density in our previous work,[24] is closely related not only to the previously little known term responsible for the diffusive flux in the rotating frame, but also to the Coriolis drift coupling to the perturbed electric field.[23]

(iv) The basic implications of scalings of the pinch velocities in relation to their underlying physics mechanisms have been discussed. In particular, we have observed that some proposed scalings must come from physics mechanisms which exist in a simpler geometry, rather than the toroidal geometric effect which is the focal point of this paper.

Several other comments are in order here. First, this work is in the spirit of identifying the most common elements from a quasilinear theory in toroidal geometry, and focuses on evaluating the momentum pinch given an absolutely minimal characterization of the turbulence. In particular, the effects of magnetic curvature coupling to ion temperature fluctuations,[24] nonlinear wave-particle interaction,[25] the residual stress from the $\mathbf{E}\times\mathbf{B}$ shear,[22] and turbulence spreading,[55–64] are not addressed here. All of these effects may contribute to non-diffusive momentum transport. Indeed, depending on plasma parameters and configurations, a specific mechanism can be more relevant than others, and sometimes a combination of two or more mechanisms would be necessary to reproduce basic features of experiments. For instance, for spontaneous core rotation of NBI-free plasmas with H-mode edge, it seems both the residual stress and an inward pinch are needed. From the ∇P_i-driven $\mathbf{E}\times\mathbf{B}$ shear in the pedestal, one can get an enhancement of edge toroidal rotation via the residual stress, while an inward pinch is needed to form a rotation profile that peaks at the axis. It is crucial to note the dual role played by the mean $\mathbf{E}\times\mathbf{B}$ shear, i.e., the reduction of turbulence and transport due to shearing,[65,66] and the production of the residual stress via symmetry breaking.[22] Of course, details depend on the edge boundary conditions and flows in the scrape-off layer.[67,68]

Outstanding issues for future theoretical research include the role of perpendicular flows in toroidal momentum transport and the dynamics of poloidal momentum transport. Both of these can be quite important, since experimental measurements of poloidal flows exhibit significant deviations from the neoclassical theory predictions.[69,70] We note that a proper gyrokinetic treatment of this problem requires not only a lengthy calculation along the lines of Ref. 71, but also a deeper understanding of the wave-particle resonance.[25]

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